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Multiple Contracts: The Cases of SIMC and SALC

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ABSTRACT:- In 1984 the Brazilian System of Housing Financing (SFH) introduced the Mixed System of Amortization with Increasing Installments (SIMC), and in 1988 Jorge Oscar de Mello Flores, then a member of the BNH Council, presented the Linearly Increasing Amortization System – SALC as a new way to increment house financing commercialization. In this article we analyze both methods of amortization and compare them with the multiple scheme system, introduced by De Losso et al. (2013), to determine, in each case, the best option for the financing institution. Additionally, an alternative to SIMC is also analyzed. Given the choice, the borrower should request the financing institution implement it.

Keywords: Mixed System of Amortization with Increasing Installments (SIMC), Linearly Increasing Amortization System (SALC), Multiple Contracts Scheme.

I. INTRODUCTION

In 1984, the Brazilian *Banco Nacional de Habitação* – BNH (National Bank of Housing), which was then responsible for the Brazilian *Sistema Financeiro de Habitação* – SFH (System of Housing Financing), and which at that time was aiming to reduce the observed backlog of houses not being able to be commercialized, introduced the so-called *Sistema Misto de Amortizações com Prestações Reais Crescentes* – SIMC (Mixed System of Amortization with Increasing Installments in Real Terms).

Although it is no longer available in the SFH, it seems to be worthwhile examining its attributes as it may be useful for financing institutions operating outside of the SFH.

On the other hand, Jorge Oscar de Mello Flores, in 1988, then a member of the BNH Council, presented the Linearly Increasing Amortization System – SALC as an alternative to the so-called System of Mixed Amortization (SAM), which was introduced by the BNH in 1979, and which, in the context of multiple contracts, was addressed in de Faro and Lachtermacher (2025).

It is worthwhile noting that Flores's (1988) original proposition was critically analyzed in de Faro (2013), in the context of a more general approach.

In this article we will address both methods, as well as the implementation of the multiple contracts scheme for both methods.

Additionally, considering a particular alternative to SIMC, a comparison is presented considering both the point of view of the borrower and that of the financial institution providing the loan.

II. SIMC – BASIC CHARACTERISTICS

Considering a loan of F units of capital (noting that at the time of the introduction of SIMC the Brazilian currency was named *Cruzeiro*), with a term of n months, with $n>24$, at the monthly compound interest rate i, it was specified that:

a) Considering the proportion of only 85% of the loan amount F, the borrower would be required to pay 24 constant monthly installments, with value $\bar{P}$, in accordance with the amortization system of constant payments, which in Brazil is called *Tabela Price* (TP) or French Method.

b) Additionally, the borrower would also have to pay, starting at the end of the 25th month, a sequence of increasing n – 24 monthly payments, following an arithmetic progression with ratio $\bar{R}$, given by:

$$\bar{R} = \frac{0{,}176470588 \times \bar{P} \times i \times \left(a_{\overline{n-23}|i} + s_{\overline{23}|i} \right)}{\left[1 + (n-23) \times i \right] \times a_{\overline{n-23}|i} - n + 23} \quad (1)$$

where according to the classical notation of mathematics of finance, cf. de Finetti (1969) and Kellison (1981), considering an annuity – immediate with n unitary payments, at the periodic rate i of compound interest, we have

$$a_{\overline{n}|i} = \left[1-(1+i)^{-n}\right]/i \tag{2}$$

and

$$s_{\overline{n}|i} = \left\{(1+i)^{n}-1\right\}/i \tag{3}$$

That is, $a_{\overline{n}|i}$ denotes the present value of the annuity, whereas $s_{\overline{n}|i}$ denotes its accumulated value at time n.

Taking into consideration the above notation, it follows that:

$$P_k = \begin{cases} \bar{P} = 0,85 \times \left(F / a_{\overline{n}|i}\right) & ,k = 1,2,...,24 \\ \bar{P} + (k-24) \times \bar{R} & ,k = 25,26,...,n \end{cases} \tag{4}$$

At this point it should be noted that the expression of $\bar{R}$, as stated by BNH, and expressed by equation (1), is very peculiar and probably not easily grasped at first. A formal derivation is presented in de Faro (1984).

Given that S_k is the outstanding balance at time *k*, immediately after the payment of installment P_k, the interest portion, I_k, associated with that installment is given by:

$$I_k = \min\left\{P_k, i \times S_{k-1}\right\} \quad ,k = 1,2,...,n \tag{5}$$

and the amortization parcel A_k, as well as the outstanding balance at time k, S_k, are given by

$$A_k = P_k - I_k \tag{6}$$

$$S_k = S_{k-1} - A_k \tag{7}$$

for $k = 1,2,\ldots,n$, with $S_0 = F$.

2.1- <u>Single Contract</u>

It should be noted that, in general, according to the usual terms of the loans financed by BNH, we have $120 \le n \le 360$ months.

Noting that the monthly interest rates in Brazil are usually less than 3%, in real terms, consider the case where F = 100,000.00 units of capital, a compound interest rate i = 1% per month and the term n of the loan is fixed at 240 months. In this case, the rate of increase of the installments after the first 24 installments is $\bar{R} = 2.9737283730$. Table 1 shows the corresponding evolution of the debt.

Table 1 – SIMC – Compound Capitalization – ***n***=240 months

k	I_k	A_k	P_k	S_k
0				100,000.00
1	1,000.00	-64.08	935.92	100,064.08
2	1,000.64	-64.72	935.92	100,128.79
3	1,001.29	-65.36	935.92	100,194.16
4	1,001.94	-66.02	935.92	100,260.18
5	1,002.60	-66.68	935.92	100,326.86
:	:	:	:	:
24	1,016.48	-80.55	935.92	101,728.37
25	1,017.28	-78.39	938.90	101,806.76
26	1,018.07	-76.20	941.87	101,882.96
:	:	:	:	:
54	1,030.27	-5.14	1,025.14	103,032.15

55	1,030.32	-2.21	1,028.11	103,034.36
56	1,030.34	0.74	1,031.08	103,033.63
57	1,030.34	3.72	1,034.06	103,029.91
58	1,030.30	6.73	1,037.03	103,023.18
:	:	:	:	:
237	61.41	1,507.92	1,569.33	4,632.80
238	46.33	1,525.97	1,572.30	3,106.83
239	31.07	1,544.21	1,575.27	1,562.62
240	15.63	1,562.62	1,578.25	0.00
∑	194,313.87	100,000,00	294,313.87	

It should be noted that up to the 55th payment, all parcels of amortization are negative resulting in an increase of the outstanding balance. After this point, all the amortizations turn positive, resulting in the decrease of the outstanding balance.

2.2- <u>The Multiple Contracts</u>

Following De-Losso et al. (2013), instead of requiring the borrower to engage in a single contract, the financing institution may decide to substitute the single contract by n multiple subcontracts, one for each of the n payments of the single contract, with the loan amount of the k^{th} subcontract, also named as principal, and denoted as F_k, being equal to the present value, at the same interest rate i of the single contract. That is:

$$F_k = P_k / (1+i)^k \quad , k = 1, 2, ..., n \tag{8}$$

Regarding the parcels of amortization, $\bar{A}_k$, and of interest, $\bar{I}_k$, of each subcontract, we have:

$$\bar{A}_k = F_k \quad , k = 1, 2, ..., n \tag{9}$$

and

$$\bar{I}_k = P_k - \bar{A}_k = P_k \times \left[1 - (1+i)^{-k}\right] \quad , k = 1, 2, ..., n \tag{10}$$

Table 2 shows the multiple contracts case with the strict adherence of the basic characteristics of the SIMC, with F = 100,000.00 units of capital, i = 1% per month and n=36 months.

Table 2 – SIMC – Compound Capitalization – ***n***=36 months Multiple Contracts

k	$F_k = \bar{A}_k$	$\bar{I}_k$	$\bar{P}_k = P_k$
1	2,795.26	27.95	2,823.22
2	2,767.59	55.63	2,823.22
3	2,740.19	83.03	2,823.22
4	2,713.06	110.16	2,823.22
5	2,686.19	137.02	2,823.22
6	2,659.60	163.62	2,823.22
7	2,633.26	189.95	2,823.22
8	2,607.19	216.02	2,823.22
9	2,581.38	241.84	2,823.22
10	2,555.82	267.40	2,823.22
11	2,530.52	292.70	2,823.22
12	2,505.46	317.76	2,823.22
:	:	:	:
24	2,223.47	599.75	2,823.22
25	2,408.23	680.16	3,088.39
26	2,589.12	764.45	3,353.57

27	2,766.19	852.56	3,618.75
28	2,939.49	944.43	3,883.92
29	3,109.10	1,040.01	4,149.10
30	3,275.06	1,139.22	4,414.28
31	3,437.42	1,242.03	4,679.46
32	3,596.25	1,348.38	4,944.63
33	3,751.60	1,458.21	5,209.81
34	3,903.52	1,571.47	5,474.99
35	4,052.06	1,688.10	5,740.17
36	4,197.28	1,808.06	6,005.34
∑	100,000.00	22,319.61	122,319.61

It should be noted that we have:

$$\sum_{k=1}^{n} I_k = \sum_{k=1}^{n} \bar{I}_k \tag{11}$$

2.3- <u>Fiscal Gain Analysis</u>

From an accounting perspective, there are no apparent differences since the total interest is the same in both cases. However, the tax gain for the entity providing the financing must be duly considered.

To show this, consider the results in Table 3, where we have the sequences of the values of I_k and of $\bar{I}_k$, as well as the sequence as differences $\Delta_k = I_k - \bar{I}_k$, for $k = 1, 2, \ldots, n.$

It is important to note that the succession of values Δ_k presents a single sign variation. Therefore, according to de Faro (1974), a conventional financing project is identified, to which a single internal rate of return is associated, which, in this case since $\sum_{k=1}^{n}\left(I_k - \bar{I}_k\right) = 0$, is zero.

Table 3- Comparison of I_k and of $\bar{I}_k$ - The Case of SIMC

k	I_k	$\bar{I}_k$	Δ_k
1	1,000.00	27.95	972.05
2	981.77	55.63	926.14
3	963.35	83.03	880.32
4	944.75	110.16	834.59
5	925.97	137.02	788.95
6	907.00	163.62	743.38
7	887.84	189.95	697.88
8	868.48	216.02	652.46
9	848.93	241.84	607.10
10	829.19	267.40	561.80
11	809.25	292.70	516.55
12	789.11	317.76	471.36
:	:	:	:
24	531.14	599.75	-68.61
25	508.22	680.16	-171.95
26	482.41	764.45	-282.04
27	453.70	852.56	-398.86
28	422.05	944.43	-522.38
29	387.43	1,040.01	-652.57

30	349.82	1,139.22	-789.41
31	309.17	1,242.03	-932.86
32	265.47	1,348.38	-1,082.91
33	218.68	1,458.21	-1,239.53
34	168.77	1,571.47	-1,402.70
35	115.70	1,688.10	-1,572.40
36	59.46	1,808.06	-1,748.60
∑	22,319.61	22,319.61	0.00

When this crucial component is considered, disregarding costs of bookkeeping and of registration of each subcontract, the conclusion is that the financial institution is always better off if a single contract is substituted by multiple contracts.

To confirm this, denoting by ρ the financing institution cost of capital, supposing the same periodicity of the interest rate i, it is sufficient to verify if the present value of the interest sequence of the single contract, denoted by $V^S(\rho)$, and given by

$$V^S(\rho) = \sum_{k=1}^{n} I_k \times (1+\rho)^{-k} \tag{12}$$

is bigger than the present value of the multiples contract scheme, denoted by $V^M(\rho)$, and given by

$$V^M(\rho) = \sum_{k=1}^{n} \overline{I}_k \times (1+\rho)^{-k} \tag{13}$$

That is, we have $V^S(\rho) > V^M(\rho)$, if $\rho > 0$. Furthermore, the percentual fiscal gain, $\delta(\%)$, is defined as:

$$\delta(\%) = 100 \times \left\{ \left[V^S(\rho) / V^M(\rho) \right] - 1 \right\} \tag{14}$$

In Tables 4, 5 and 6, ρ_a, when it is identified as the annual equivalent value of the rate ρ, and the financing terms are expressed in annual terms, show that the values of the fiscal gain are very significant.

Table 4 – SIMC – Compound Capitalization – $\delta(\%)$ – *i*=0.5%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	8.376	17.035	25.953	35.103	44.457	53.990
10	16.407	34.507	54.177	75.247	97.520	120.780
15	24.165	52.257	83.879	118.417	155.144	193.319
20	31.550	69.727	113.460	161.161	211.157	262.014
25	38.496	86.420	141.425	200.448	260.809	320.698
30	44.955	101.925	166.633	234.388	301.934	367.635

Table 5 – SIMC – Compound Capitalization – $\delta(\%)$ – *i*=1.0%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	7.972	16.188	24.624	33.254	42.052	50.994
10	14.794	30.917	48.237	66.591	85.797	105.665
15	20.617	43.990	69.698	97.193	125.891	155.243
20	25.481	55.106	87.859	122.525	157.979	193.356
25	29.484	64.223	102.300	141.723	181.007	219.299
30	32.742	71.475	113.184	155.272	196.273	235.627

Table 6 – SIMC – Compound Capitalization – $\delta(\%)$ – *i*=2.0%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	7.232	14.644	22.214	29.918	37.734	45.642
10	12.128	25.095	38.777	53.037	67.730	82.717
15	15.400	32.262	50.241	68.952	88.026	107.146
20	17.556	37.002	57.653	78.812	99.917	120.584
25	18.985	40.089	62.231	84.459	106.168	127.073
30	19.950	42.092	64.963	87.495	109.192	129.926

In other words, the financial institution should opt to replace a single contract with multiple contracts, since $V^S(\rho) > V^M(\rho)$ for all the cases studied.

2.4 An Alternative

In the previous section strict adherence to the SIMC was addressed considering both the single and the multiples contracts options. In this section, we will analyze what can be interpreted as an alternative to SIMC, denoted by SIMCH.

Specifically, suppose that the fraction $\alpha \times F$ of loan *F* must be reimbursed by n' constant payments with the remaining fraction $(1-\alpha)\times F$ being repaid by a distinct sequence of $n-n'$ constant payments.

Denoting by *P* the value of the first n' payments, we will have:

$$P = \alpha \times F \times i / \left\{1-(1+i)^{-n'}\right\} \tag{15}$$

While, denoting as P' the value of the remaining $n-n'$ payments, it follows that:

$$P' = (1-\alpha)\times F \times (1+i)^{n'} \times i / \left\{1-(1+i)^{n'-n}\right\} \tag{16}$$

$0 < \alpha < 1.$ with the sequence of the $n-n'$ constant payments P' starting at the end of period $n'+1$.

It should be noted that this alternative, even if $n' = 24$ periods, is not a generalization of the SIMC since the remaining $n-24$ payments are supposed to be constant and not increasing in accordance with an arithmetic progression.

Fixing $n' = 24$ periods, it is worth considering, from the point of view of the borrower, to compare the totals of the payments of interest and of installments, in the case of this alternative, with the same totals in the case of SIMC.

To this end, the borrower should require the lender to fix the fraction α to be such that:

$$\alpha = 0.85 \times \left\{1-(1+i)^{-24}\right\} / \left\{1-(1+i)^{-n}\right\} \tag{17}$$

for the first 24 installments.

It should be noted that the first 24 installments will be exactly equal to the SIMC method. So, *P* is given by:

$$P = F \times i / \left\{1-(1+i)^{-n}\right\} \times 0,85 \tag{18}$$

Tables 7 and 8 show the total interest and the total installments paid by a client to the financing institution for a loan *F* = 100,000 units of capital, using several interest rates and terms, as well as the SIMC method.

Table 7 – SIMC – Compound Capitalization – Total Interest Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	17,668.66	37,620.44	60,054.09	85,200.84	113,337.41	144,801.16
10	36,392.19	81,516.79	136,384.96	202,264.32	280,879.65	374,591.00
15	57,066.37	133,703.61	232,499.78	357,051.01	512,653.16	706,558.76
20	79,727.72	194,313.87	348,755.62	550,662.71	811,167.58	1,144,700.49
25	104,398.16	263,358.84	485,174.43	783,099.04	1,175,677.83	1,686,222.17

30	131,087.06	340,777.36	641,519.26	1,053,449.66	1,603,868.97	2,327,256.97

Table 8 – SIMC – Compound Capitalization – Total Installments Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	117,668.66	137,620.44	160,054.09	185,200.84	213,337.41	244,801.16
10	136,392.19	181,516.79	236,384.96	302,264.32	380,879.65	474,591.00
15	157,066.37	233,703.61	332,499.78	457,051.01	612,653.16	806,558.76
20	179,727.72	294,313.87	448,755.62	650,662.71	911,167.58	1,244,700.49
25	204,398.16	363,358.84	585,174.43	883,099.04	1,275,677.83	1,786,222.17
30	231,087.06	440,777.36	741,519.26	1,153,449.66	1,703,868.96	2,427,256.97

Tables 9 and 10 show the same totals if the SIMCH method is adopted.

Table 9 – SIMCH – Compound Capitalization – Total Interest Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	17,114.10	36,220.30	57,413.65	80,792.56	106,465.28	134.557,32
10	34,570.90	76,057.35	124,388.82	179,350.51	240,715.39	308.356,63
15	53,500.17	121,320.06	202,022.41	293,782.14	395,150.51	505.390,36
20	73,840.01	171,189.33	287,213.00	417,244.93	558,581.96	710.429,88
25	95,508.22	224,743.42	377,198.73	545,089.17	725,295.61	917.653,13
30	118,406.95	281,093.11	469,969.75	674,830.72	893,061.78	1.125.395,36

Table 10 – SIMCH – Compound Capitalization – Total Installments Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	117,114.10	136,220.30	157,413.65	180,792.56	206,465.28	234,557.32
10	134,570.90	176,057.35	224,388.82	279,350.51	340,715.39	408,356.63
15	153,500.17	221,320.06	302,022.41	393,782.14	495,150.51	605,390.36
20	173,840.01	271,189.33	387,213.00	517,244.93	658,581.96	810,429.88
25	195,508.22	324,743.42	477,198.73	645,089.17	825,295.61	1,017,653.13
30	218,406.95	381,093.11	569,969.75	774,830.72	993,061.78	1,225,395.36

$\beta(\%)$ denotes the percentage difference of the values of interest and installments given both methods by:

$$\beta(\%) = \frac{V^{SIMC} - V^{SIMCH}}{V^{SIMCH}} \times 100 \quad (19)$$

where V represents the interest or the installment of each method in Tables 11 and 12, and 13 and 14 present the corresponding values,

Table 11 – β (%) – Total Interest Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	3.24%	3.87%	4.60%	5.46%	6.45%	7.61%
10	5.27%	7.18%	9.64%	12.78%	16.69%	21.48%
15	6.67%	10.21%	15.09%	21.54%	29.74%	39.80%
20	7.97%	13.51%	21.43%	31.98%	45.22%	61.13%
25	9.31%	17.18%	28.63%	43.66%	62.10%	83.75%
30	10.71%	21.23%	36.50%	56.11%	79.59%	106.79%

Table 12 – β (%) – Total Installments Paid

	Interest rate					
n(years)	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
5	0.47%	1.03%	1.68%	2.44%	3.33%	4.37%
10	1.35%	3.10%	5.35%	8.20%	11.79%	16.22%
15	2.32%	5.60%	10.09%	16.07%	23.73%	33.23%
20	3.39%	8.53%	15.89%	25.79%	38.35%	53.59%
25	4.55%	11.89%	22.63%	36.90%	54.57%	75.52%
30	5.81%	15.66%	30.10%	48.86%	71.58%	98.08%

which are also shown in Graph 1 and Graph 2,

Figure 1 – β (%) Interest Paid

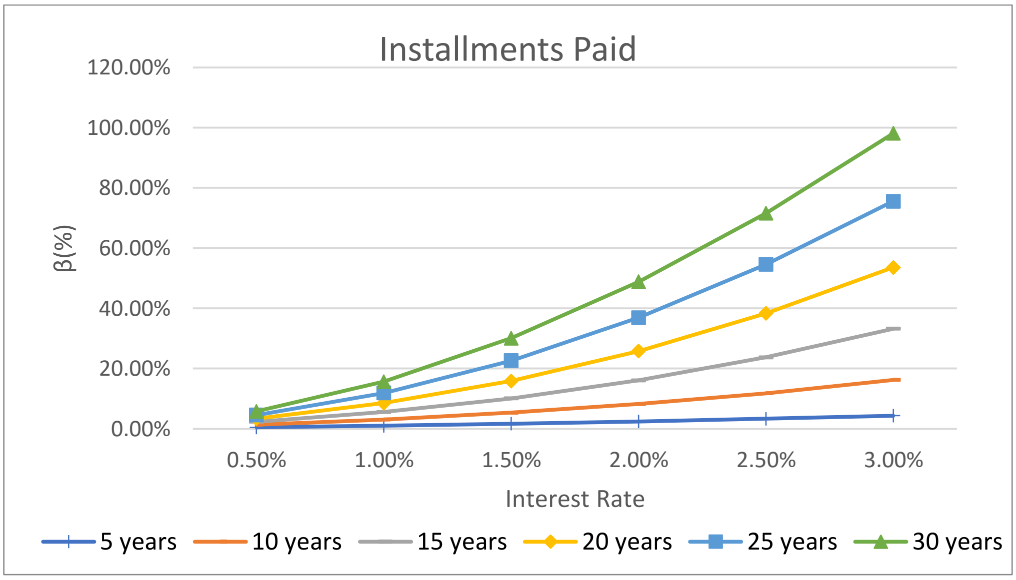

Figure 2 – β (%) Installment Paid

The results show that the borrower is always better off if the alternative option is implemented.

2.4.1- Point of View of the Lender

Supposing that the borrower has determined that the loan should be in accordance with the alternative, the lender has the option of choosing either the single or the multiple contracts version.

Extending the analysis of section 2.2, with $V^S(\rho)$ and $V^M(\rho)$ representing, respectively, the present values of the single and multiple contracts when the alternative is implemented, Tables 13, 14 and 15 present the corresponding values of the percentual fiscal gain δ.

Table 13 – SIMC Hybrid – Compound Capitalization – $\delta(\%)$ – *i*=0.5%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	8,113	16,471	25,050	33,828	42,778	51,880
10	15,792	33,033	51,584	71,279	91,937	113,369
15	23,168	49,639	78,953	110,519	143,703	177,911
20	30,122	65,694	105,546	148,248	192,460	237,134
25	36,584	80,722	130,028	181,943	234,499	286,501
30	42,514	94,370	151,514	210,300	268,545	325,297

Table 14 – SIMC Hybrid – Compound Capitalization – $\delta(\%)$ – *i*=1.0%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	7,707	15,620	23,718	31,977	40,375	48,891
10	14,177	29,453	45,688	62,729	80,414	98,587
15	19,637	41,468	65,051	89,891	115,514	141,508
20	24,116	51,378	80,785	111,330	142,202	172,844
25	27,722	59,211	92,712	126,744	160,378	193,160
30	30,584	65,178	101,237	137,041	171,825	205,386

Table 15 – SIMC Hybrid – Compound Capitalization – $\delta(\%)$ – *i*=2.0%p.m.

	ρ_a(%)					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	6,963	14,070	21,301	28,636	36,057	43,546
10	11,516	23,661	36,316	49,358	62,672	76,150
15	14,480	29,951	46,087	62,580	79,172	95,669
20	16,371	33,868	51,895	69,974	87,793	105,183
25	17,586	36,251	55,147	73,737	91,779	109,211
30	18,385	37,691	56,896	75,526	93,470	110,765

As shown, the values of δ are always positive and very significant. Therefore, if the lender specifies that the alternative must be implemented, the borrower should require the adoption of the multiple contracts option.

3- SALC – Basic Characteristics

Suggested as an alternative to the SAM Method, which was examined in de Faro and Lachtermacher (2025), Jorge Oscar de Mello Flores presented to the BNH Council what he named the Linearly Increasing Amortization System – SALC. In what follows, considering what was discussed in de Faro (2013), Flores's original suggestion will be analyzed.

Considering the loan amount *F*, which must be redeemed by paying *n* periodic installments, at the effective periodic rate *i* of compound interest, assume P_k to be the k^{th} installment, and assume I_k and A_k to be the respective interest and amortization parcels. As usual, we have that:

$$P_k = I_k + A_k \quad ; \; k = 1, 2, \ldots, n \tag{20}$$

where

$$I_k = \begin{cases} i \times F = i \times S_0 & ; \quad k = 1 \\ i \times S_{k-1} = i \times \left\{ F - \sum_{\ell=1}^{k-1} A_\ell \right\} & ; \quad k = 2, 3, \ldots, n \end{cases} \tag{21}$$

and S_k, for $k = 1, 2, \ldots, n$, denote the outstanding balance immediately after the k^{th} payment with $S_0 = F$.

Suppose now that the amortization parcels form an arithmetic progression with the initial term is:

$$A_1 = \gamma \times F / n \quad ; \; \alpha > 0 \tag{22}$$

so that:

$$A_k = \gamma \times F / n + (k-1) \times R \quad ; \; k = 1, 2, \ldots, n \tag{23}$$

where the ratio is given by:

$$R = 2 \times (1-\gamma) \times F / \{ n \times (n-1) \} \tag{24}$$

Note that there will be increasing amortization parcels, which is Flores's proposition, if $\gamma < 1$. If $\gamma = 1$, we have the case of the constant amortization method.

Regarding the outstanding balance, S_k, we have:

$$S_k = F - \sum_{\ell=1}^{k} A_\ell = F - \left[(A_1 + A_k) \times k \right] / 2 \tag{25}$$

or

$$S_k = F \times \left\{ 1 - \left[(n \times \gamma - 1) \times k + (1-\gamma) \times k^2 \right] \right\} / \{ n \times (n-1) \}, \; k = 0, 1, \ldots, n \tag{26}$$

As for the behavior of the payment, P_k, note that, starting from relation (15), we have:

$$P_k = i \times S_{k-1} + A_k \tag{27}$$

Therefore, considering relations (23), (24) and (25), it follows that:

$$P_k = F \times \{ \gamma \times (n-1) + 2 \times (1-\gamma) \times (k-1) + i \times \left[n \times (n-1) - (n \times \gamma - 1) \times (k-1) - (1-\gamma) \times (k-1)^2 \right] \} / \{ n \times (n-1) \}; \; k = 1, 2, \ldots, n \tag{28}$$

which is a rather complex expression but is easily solved numerically.

3.1- <u>Single Contract</u>

Considering F = 100,000.00 units of capital, α=0.7, a compound interest rate i=1% per month and a term n = 36 months, the evolution of the outstanding debt is shown in Table 16.

Table 16 – SALC $(\gamma = 0.7)$ – Compound Capitalization – ***n***=36 months

k	I_k	A_k	P_k	S_k
0				100,000.00
1	1,000.00	1,944.44	2,944.44	98,055.56
2	980.56	1,992.06	2,972.62	96,063.49
3	960.63	2,039.68	3,000.32	94,023.81
4	940.24	2,087.30	3,027.54	91,936.51
5	919.37	2,134.92	3,054.29	89,801.59
6	898.02	2,182.54	3,080.56	87,619.05
7	876.19	2,230.16	3,106.35	85,388.89
8	853.89	2,277.78	3,131.67	83,111.11
9	831.11	2,325.40	3,156.51	80,785.71
10	807.86	2,373.02	3,180.87	78,412.70
11	784.13	2,420.63	3,204.76	75,992.06
12	759.92	2,468.25	3,228.17	73,523.81

:	:	:	:	:
24	432.30	3,039.68	3,471.98	40,190.48
25	401.90	3,087.30	3,489.21	37,103.17
26	371.03	3,134.92	3,505.95	33,968.25
27	339.68	3,182.54	3,522.22	30,785.71
28	307.86	3,230.16	3,538.02	27,555.56
29	275.56	3,277.78	3,553.33	24,277.78
30	242.78	3,325.40	3,568.17	20,952.38
31	209.52	3,373.02	3,582.54	17,579.37
32	175.79	3,420.63	3,596.43	14,158.73
33	141.59	3,468.25	3,609.84	10,690.48
34	106.90	3,515.87	3,622.78	7,174.60
35	71.75	3,563.49	3,635.24	3,611.11
36	36.11	3,611.11	3,647.22	0.00
$\sum$	20,350.00	100,000.00	120.350.00	

3.2- <u>Multiple Contracts</u>

Like the case of SIMC, instead of requiring the borrower to engage in a single contract, the financing institution may decide to substitute the single contract by *n* multiple subcontracts, one for each of the *n* payments of the single contract.

Considering that relations (8), (9) and (10), which refer to the case of SIMC, are also applicable in the case of SALC, Table 17 presents the corresponding values.

Table 17 – SALC $(\gamma = 0.7)$ – Compound Capitalization – ***n***=36 months

Multiple Contracts			
k	$F_k = \bar{A}_k$	$\bar{I}_k$	$\bar{P}_k = P_k$
1	2,915.29	29.15	2,944.44
2	2,914.05	58.57	2,972.62
3	2,912.08	88.24	3,000.32
4	2,909.41	118.13	3,027.54
5	2,906.05	148.24	3,054.29
6	2,902.02	178.53	3,080.56
7	2,897.35	209.00	3,106.35
8	2,892.04	239.63	3,131.67
9	2,886.12	270.39	3,156.51
10	2,879.60	301.27	3,180.87
11	2,872.50	332.26	3,204.76
12	2,864.84	363.33	3,228.17
:	:	:	:
24	2,734.42	737.57	3,471.98
25	2,720.77	768.43	3,489.21
26	2,706.76	799.19	3,505.95
27	2,692.40	829.82	3,522.22
28	2,677.70	860.32	3,538.02
29	2,662.66	890.67	3,553.33
30	2,647.31	920.86	3,568.17
31	2,631.65	950.89	3,582.54
32	2,615.70	980.73	3,596.43

33	2,599.46	1,010.38	3,609.84
34	2,582.94	1,039.83	3,622.78
35	2,566.17	1,069.07	3,635.24
36	2,549.13	1,098.09	3,647.22
∑	100,000.00	20,350.00	120,350.00

It should be noted that like the case of SIMC, we also have $\sum_{k=1}^{36} I_k = \sum_{k=1}^{36} \overline{I}_k = 20,350.00$.

3.3- Fiscal Gain Analysis

Analogous to the case of SIMC, even though the interest total is the same in both cases, single and multiple contracts, the tax gain for the entity providing the financing must be duly considered.

As in the case of SIMC, the first step is to compare the corresponding sequences of interest parcels and their differences, as shown in Table 18.

Table 18 – Comparison of I_k and $\overline{I}_k$ - The Case of SALC

k	I_k	$\overline{I}_k$	Δ_k
1	1,000.00	29.15	970.85
2	980.56	58.57	921.98
3	960.63	88.24	872.40
4	940.24	118.13	822.10
5	919.37	148.24	771.13
6	898.02	178.53	719.48
7	876.19	209.00	667.19
8	853.89	239.63	614.26
9	831.11	270.39	560.72
10	807.86	301.27	506.59
11	784.13	332.26	451.87
12	759.92	363.33	396.59
:	:	:	:
24	432.30	737.57	-305.27
25	401.90	768.43	-366.53
26	371.03	799.19	-428.16
27	339.68	829.82	-490.14
28	307.86	860.32	-552.46
29	275.56	890.67	-615.12
30	242.78	920.86	-678.09
31	209.52	950.89	-741.36
32	175.79	980.73	-804.94
33	141.59	1,010.38	-868.80
34	106.90	1,039.83	-932.93
35	71.75	1,069.07	-997.33
36	36.11	1,098.09	-1,061.98
∑	20,350.00	20,350.00	0.00

Relative to the case of SIMC, the sequence of differences $\Delta_k = I_k - \overline{I}_k$, also characterizes a conventional financing project, in which the internal rate of return is unique and equal to zero. Consequently, we can additionally conclude, in the case of SALC, that $V^S(\rho) > V^M(\rho)$, if $\rho > 0$.

Therefore, also in the case of SALC, the financing institution providing the loan is better off substituting a single contract by subcontracts, one for each payment of the single contract.

This result is corroborated by the data in Tables 19, 20, and 21.

Table 19 – SALC $(\gamma = 0.7)$ – Compound Capitalization – $\delta(\%)$ – *i*=0.5%p.m.

	$\rho_a(\%)$					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	8.067	16.360	24.854	33.523	42.341	51.284
10	15.618	32.589	50.752	69.925	89.909	110.509
15	22.479	47.825	75.516	104.935	135.464	166.553
20	28.640	61.640	97.769	135.708	174.327	212.804
25	34.121	73.817	116.837	161.118	205.229	248.365
30	38.964	84.324	132.663	181.386	229.090	275.199

Table 20 – SALC $(\gamma = 0.7)$ – Compound Capitalization – $\delta(\%)$ – *i*=1.0%p.m.

	$\rho_a(\%)$					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	7.462	15.076	22.815	30.655	38.576	46.554
10	13.457	27.727	42.649	58.061	73.806	89.740
15	18.155	37.761	58.366	79.537	100.903	122.176
20	21.835	45.538	70.247	95.240	120.007	144.235
25	24.732	51.510	79.039	106.413	133.129	158.962
30	27.032	56.094	85.524	114.375	142.252	169.047

Table 21 – SALC $(\gamma = 0.7)$ – Compound Capitalization – $\delta(\%)$ – *i*=2.0%p.m.

	$\rho_a(\%)$					
n(years)	5.00%	10.00%	15.00%	20.00%	25.00%	30.00%
5	6.461	12.972	19.514	26.068	32.617	39.147
10	10.423	21.134	32.016	42.966	53.898	64.741
15	12.905	26.210	39.662	53.055	66.245	79.137
20	14.537	29.479	44.430	59.127	73.420	87.239
25	15.664	31.670	47.515	62.926	77.789	92.077
30	16.475	33.196	49.593	65.421	80.613	95.180

IV. CONCLUSION

As previously shown, using compound capitalization, by De-Losso et al. (2013) for the case of amortization with constant payments, by de Faro (2022), for the case of a constant parcel of amortization, and by de Faro and Lachtermacher (2023a) for the SACRE system of amortization, a financial institution providing a loan should always implement a multiple contracts option rather than that of a single contract.

The same results were shown using simple capitalization, by de Faro & Lachtermacher (2023b) for the case of amortization with constant payments, and by Lachtermacher & de Faro (2024a and 2024b) for the SACRE-F Method and German Method.

Regarding the proposed alternative to the SIMC, named SIMCH, it was shown that, considering the point of view of the borrower, the best option for the borrower is to require the financing institution to implement it.

On the other hand, considering now the point of view of the financing institution providing the loan, the best option is to implement the multiple contracts version.

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